

# Universal fermionic spectral functions from string theory

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We carry out the first holographic calculation of a fermionic response function for a strongly coupled  $d = 3$  system with an explicit  $D = 10$  or  $D = 11$  supergravity dual. By considering the supersymmetry current, we obtain a universal result applicable to all  $d = 3$   $N = 2$  SCFTs with such duals. Surprisingly, the spectral function does not exhibit a Fermi surface, despite the fact that the system is at finite charge density. We show that it has a phonino pole and at low frequencies there is a depletion of spectral weight with a power-law scaling which is governed by a locally quantum critical point.

## INTRODUCTION

The AdS/CFT correspondence provides a powerful framework for studying strongly coupled quantum field theories and has recently been used as a theoretical laboratory for studying condensed matter systems. Some of the most interesting “AdS/CMT” studies have focussed on calculating fermionic response functions, not least in the hope of obtaining a better understanding of the deeply vexing non-fermi liquids that are seen in a variety of materials at finite charge density including the heavy fermion and high- $T_c$  cuprate superconductors.

The ground-breaking works [1–4] gave the first such “holographic” calculations of fermion spectral functions using phenomenological or “bottom-up” models. The AdS/CFT correspondence states that certain classes of field theories have specific dual gravitational descriptions, determined by their realisation in string theory. In the bottom-up approach, rather than identify a specific string dual, one simply postulates a particular theory of gravity with some simple matter content and couplings and assumes that it captures the essential features of potential dual field theories. Specifically the original papers considered the Dirac equation for a minimally coupled spin 1/2 fermion with mass  $m$  and charge  $q$  in the gravitational background of a four-dimensional AdS-Reissner-Nordström ( $AdS_4$ -RN) black-brane. With appropriate boundary conditions, this encodes a fermionic response function at finite temperature and chemical potential. It was shown that the resulting spectral function can exhibit a Fermi surface with non-Fermi liquid scaling for certain values of  $m$  and  $q$ . It can also have an interesting oscillatory behaviour, periodic in the logarithm of the frequency. While the existence of these Fermi surfaces depend both on the full  $AdS_4$ -RN geometry, it was shown in [4] (see also [5]), how their low-frequency scaling behaviour can be beautifully understood as a consequence of the  $AdS_2 \times \mathbb{R}^2$  “IR” region of the spacetime that is dual to an emergent one-dimensional conformal field theory (CFT).

The validity of phenomenological models rests on the hope that either somewhere in the landscape of string

theory backgrounds the model will be realised exactly, and hence the holographic calculations relate to a specific dual field theory, or alternatively, the gravitational model may only be realised approximately but the features are sufficiently robust to capture properties of some actual field theory. Although significantly more difficult, it is clearly essential to study “top-down” models in which one is carrying out holographic calculations within a explicit string theory setting and hence obtaining results for *bona fide* dual field theories.

The purpose of this letter is to communicate the first such calculations of fermion spectral functions in ten- or eleven-dimensional supergravity, the low-energy limit of string/M-theory. The most robust and controlled examples of holography are for supersymmetric conformal field theories (SCFTs) and we will restrict our considerations to this class. Remarkably, as we will explain, our results will be valid not just for a single field theory but for an *infinite* number.

We analyse the response function of the *universal* spin- $\frac{3}{2}$  supersymmetry current, or “supercurrent”, in the *general* infinite class of  $d = 3$ ,  $N = 2$  SCFTs that have dual gravitational backgrounds of the form  $AdS_4 \times M$  in either  $D = 10$  or  $D = 11$  supergravity. The supercurrent, the energy-momentum tensor and the global abelian  $R$ -symmetry current of the SCFT comprise a supermultiplet. It is possible to isolate this universal sector from all other operators because, from the gravitational point of view, given a Kaluza-Klein (KK) reduction of  $D = 10$  or  $D = 11$  supergravity on any appropriate manifold  $M$  one can then consistently truncate an infinite tower of fields leaving minimal  $N = 2$   $D = 4$  gauged supergravity [6]. The field content of this gauged supergravity consists of a metric, a gauge field and a Dirac gravitino, which are precisely dual to the energy-momentum tensor, the global abelian  $R$ -symmetry current and the fermionic supercurrent of the SCFT, respectively.

We consider the electrically charged  $AdS_4$ -RN black-brane solution which provides the dual description of the SCFTs at finite temperature  $T$  and chemical potential  $\mu$  with respect to the global  $R$ -symmetry, both of which break the supersymmetry. It is possible that the SCFT

undergoes a phase transition at some critical temperature  $T_c$ , which will involve other KK fields, and if it does then the  $AdS_4$ -RN description will be valid only for temperatures above  $T_c$ . It is an open question whether or not there are SCFTs which do not have such phase transitions and hence are described by the extremal  $AdS_4$ -RN black-brane all the way down to  $T = 0$ .

We calculate the supercurrent response function by solving the linearised gravitino equations in the  $AdS_4$ -RN background, as a function of frequency  $\omega$  and momentum  $k \equiv |\mathbf{k}|$ . We find that there is no log-periodic behaviour, in contrast to the bottom-up model results. Furthermore, it does *not* have a Fermi surface, i.e. a quasi-particle pole with  $\omega = 0$  and  $k \neq 0$ , as one might have expected for matter at finite charge density [7]. This surprising result underscores the importance of the top-down approach. Further study will be required to determine whether a Fermi surface will be seen in different response functions or whether they are in fact absent in these holographic theories.

The spectral function has other interesting features. It has a “phonino pole” [10–12] located at  $\omega + \mu = 0$  and  $k = 0$ , reflecting the broken supersymmetry. We also find a depletion of spectral weight at low frequencies, as seen in [13, 14], where bulk dipole couplings were considered in a bottom-up context. In [13] this behaviour was interpreted as a dynamical gap dual to something akin to a Mott insulator. A subsequent discussion of this interpretation can be found in [14]. Here we will show that at zero temperature the spectral function vanishes when  $\omega = 0$ . Furthermore, the low-frequency behaviour is weakly gapped (and thus unlike a Mott gap) and determined by an emergent one-dimensional, “locally quantum critical”, CFT, dual to the IR  $AdS_2 \times \mathbb{R}^2$  part of the geometry. This behaviour persists, albeit in a softened way, for non-zero temperatures.

In [15] we present more details of the rather technical calculations as well as some additional results.

## SUPERCURRENT RESPONSE FUNCTION

Let  $S_\alpha$  be the conserved supercurrent operator of the  $d = 3$  SCFT. It is a complex vector-spinor, where  $\alpha$  is the vector index, has conformal dimension  $\Delta = \frac{5}{2}$ , and is charged under the global  $R$ -symmetry. We will calculate the retarded correlation function  $G_{\alpha\beta}(p) = \langle S_\alpha(p) \bar{S}_\beta(0) \rangle_{Ret}$  at finite temperature and chemical potential, exploiting the fact that the expectation value of the supercurrent in the presence of a vector-spinor source  $a_\alpha$ , at linearised order, is given by

$$\langle S_\alpha \rangle = i G_{\alpha\beta} a^\beta. \quad (1)$$

The supercurrent is conserved and, because we have an SCFT, gamma-traceless:  $p^\alpha \langle S_\alpha \rangle = \gamma^\alpha \langle S_\alpha \rangle = 0$  where

$\gamma^\alpha$  are  $d = 3$  gamma-matrices. Since we are considering the SCFT at finite  $\mu$ , which can be viewed as weakly gauging the  $R$ -symmetry, we have  $p^\alpha = (\tilde{\omega}, \mathbf{k})$  with  $\tilde{\omega} \equiv \omega + \mu$ . The source can be taken to satisfy

$$\gamma^\alpha a_\alpha = 0, \quad \delta a_\alpha = (\delta_\alpha^\beta - \frac{1}{3} \gamma_\alpha \gamma^\beta) p_\beta \epsilon, \quad (2)$$

where the second equation arises from the weak gauging of the supersymmetry. Of course the supercurrent itself, and hence its expectation value, is gauge invariant.

The four independent components of  $G_{\alpha\beta}$  can be extracted by introducing a basis of 3d vector-spinors  $e_\alpha^{(i)}$ ,  $i = 1, 2$ , satisfying  $\gamma^\alpha e_\alpha^{(i)} = p^\alpha e_\alpha^{(i)} = 0$  and the normalisation condition  $\bar{e}_\alpha^{(i)} e^{(j)\alpha} = -2p^2 \epsilon^{(i)(j)}$ . We can then write  $G_{\alpha\beta} = t_{ij} e_\alpha^{(i)} \bar{e}_\beta^{(j)}$ , where the  $t_{ij}$  are the four independent components of  $G_{\alpha\beta}$ . The  $d = 3$  SCFT is invariant under spatial rotations and parity. We can use this to choose  $p^\alpha = (\tilde{\omega}, k, 0)$ , where  $k \equiv |\mathbf{k}|$ , and show that  $t_{12} = t_{21} = 0$  and  $t_{22}(\omega, k) = t_{11}(\omega, -k)$ . Thus the correlation function is determined by a single function  $t_{11}$ . Our objective is to calculate  $t_{11}(\omega, k)$ , and more specifically the spectral function,  $A(\omega, k)$ , defined as

$$A(\omega, k) \equiv \text{Im } t_{11}(\omega, k). \quad (3)$$

## HOLOGRAPHIC CALCULATION

### $N = 2$ gauged supergravity in $D = 4$

The field content of minimal  $N = 2$  gauged supergravity in  $D = 4$  [16, 17] consists of a metric  $g_{\mu\nu}$ , a gauge field  $\mathcal{A}_\mu$  and a single Dirac gravitino  $\psi_\mu$ . This theory admits the  $AdS_4$ -RN black-brane solution given by

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + \frac{r^2}{\ell^2} (dx^2 + dy^2), \quad \mathcal{A} = \phi dt, \quad (4)$$

with

$$f = \frac{r^2}{\ell^2} - \frac{r_+}{r} \left( \frac{r_+^2}{\ell^2} + \ell^2 \mu^2 \right) + \ell^2 \mu^2 \frac{r_+^2}{r^2} \quad (5)$$

$$\phi = \mu \ell \left( 1 - \frac{r_+}{r} \right).$$

The location of the horizon is  $r = r_+$ . The temperature of the black-brane is given by  $T = (3r_+/\ell^2 - \ell^2 \mu^2/r_+)/4\pi$ . When  $T = 0$ , as  $r \rightarrow r_+$  the black-brane solution approaches  $AdS_2 \times \mathbb{R}^2$  with the radius of the  $AdS_2$  given by  $L_{(2)} = \ell/\sqrt{6}$ .

We will study the equation of motion of the gravitino at the linearised level in the  $AdS_4$ -RN background (4),(5). One can use the local supersymmetry to fix the gauge  $D^\mu \psi_\mu = \Gamma^\mu \psi_\mu = 0$ , where  $\Gamma^\mu$  are  $D = 4$  gamma-matrices, and we then obtain

$$(\not{D} - m \mathbb{1} - \frac{1}{2} i \mathcal{F}^{\mu\nu} \Gamma_{\mu\nu}) \psi_\rho + i \mathcal{F}^{\mu\nu} \Gamma_\mu \Gamma_\rho \psi_\nu = 0, \quad (6)$$

where  $D \equiv \nabla - iq\mathcal{A}$ ,  $\mathcal{F} = d\mathcal{A}$  and  $q = -m = \frac{1}{\ell}$ . There are residual gauge transformations, which we fix later. We note the presence of Pauli terms.

### Asymptotic behaviour of solutions

Under the AdS/CFT correspondence, to calculate  $G_{\alpha\beta}$  one solves the linearised equations of motion for the dual gravitino field in  $AdS_4$ -RN, imposing ingoing boundary conditions at the horizon, and studies the asymptotic expansion of the field as  $r \rightarrow \infty$ . This describes a linearised perturbation of the CFT, where the  $r^{\Delta-3}$  term encodes the source  $a_\alpha$  and the  $r^{-\Delta}$  term encodes the resulting expectation value of the operator  $\langle S_\alpha \rangle$ .

We will assume throughout that the time and space dependence of the gravitino is given by  $e^{-i\omega t + i\mathbf{k} \cdot \mathbf{x}}$ . As  $r \rightarrow \infty$ , schematically, we have

$$\psi = r^{-1/2}\psi_{-1/2} + r^{-3/2}\psi_{-3/2} + r^{-5/2}\psi_{-5/2} + r^{-7/2}\psi_{-7/2} + r^{-7/2}\log r\phi_{-7/2} + \dots, \quad (7)$$

where  $\psi_{-1/2}$  etc are functions of three-momentum  $p^{\hat{\alpha}} = (\tilde{\omega}, k, 0)$ . There is an analogous expansion for the residual gauge transformations, fixed by two parameters  $\varepsilon_{1/2}$  and  $\varepsilon_{-7/2}$  appearing at orders  $r^{1/2}$  and  $r^{-7/2}$  respectively.

It is then natural to decompose all these components under the asymptotic  $d = 3$  Lorentz symmetry that appears as  $r \rightarrow \infty$ . Using that gauge conditions and residual gauge transformations, one finds that the solution is completely determined by a pair of  $d = 3$  vector-spinors  $a_\alpha$  and  $b_\alpha$  satisfying

$$\begin{aligned} \gamma^\alpha a_\alpha &= 0, & \delta a_\alpha &= \left(\delta_\alpha^\beta - \frac{1}{3}\gamma_\alpha\gamma^\beta\right)p_\beta\epsilon, \\ \gamma^\alpha b_\alpha &= 0, & p^\alpha b_\alpha &= 0, & \delta b_\alpha &= 0, \end{aligned} \quad (8)$$

where  $\epsilon$  is a  $d = 3$  spinor that determines  $\varepsilon_{1/2}$ . To order  $r^{-3/2}$  the expansion is completely determined by  $a_\alpha$ . Terms in  $a_\alpha$  also appear at order  $r^{-5/2}$  and this leads to some ambiguity in defining the new independent data  $b_\alpha$ , that appears at this order. However, it can be fixed uniquely using the second set of conditions in (8). The full expansion requires the introduction of another spinor in  $\psi_{-7/2}$ , but this data can be gauged away using  $\varepsilon_{-7/2}$ .

Since the supercurrent is a  $\Delta = 5/2$  operator in the dual  $d = 3$  SCFT, the source is fixed by the  $r^{-1/2}$  expansion data and the expectation value by the  $r^{-5/2}$  expansion data, and hence can be identified with  $a_\alpha$  and  $b_\alpha$ , respectively. Furthermore, (1) allows us to write  $b_\alpha = iG_{\alpha\beta}a^\beta$  and we can show that

$$t_{11} = -\frac{i\bar{e}_\alpha^{(2)}b^\alpha}{2p^2\bar{e}_\beta^{(1)}a^\beta}. \quad (9)$$

This is invariant under residual gauge transformations.

### Solving the gravitino equation

A convenient way to solve the gravitino equation (6) in the  $AdS_4$ -RN background and impose the ingoing bound-

ary conditions, is to dimensionally reduce on the two spatial directions  $x, y$  and decompose into  $Spin(1, 1)$  representations. Subject to the gauge conditions  $\Gamma^\mu\psi_\mu = D^\mu\psi_\mu = 0$  there are 8 independent complex components in  $\psi_\mu$ . After dimensional reduction these can be written in terms of functions of  $r$  labeled  $u^{(s)}$  and  $v^{(s)}$ , where  $s = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$  refer to the helicity of the  $Spin(1, 1)$  representation. The two sets of functions,  $u^{(s)}$  and  $v^{(s)}$ , are parity eigenstates and map into each other under a rotation by  $\pi$  in the  $x, y$  plane.

Using this decomposition the gravitino equations (6) in the  $AdS_4$ -RN background (4) are equivalent to a system of linear ODEs for  $u^{(s)}$  and  $v^{(s)}$ . The parity and rotational symmetries imply that the ODEs for  $u^{(s)}$  and  $v^{(s)}$  do not mix and map into to each other if one replaces  $k \rightarrow -k$ . At finite temperature the horizon at  $r = r_+$  is a regular singular point of the ODEs and we can develop a Frobenius expansion. Writing  $\delta_s(\omega) = s + \frac{i\omega}{2\pi T}$  the solutions have leading-order behaviour

$$\begin{aligned} u^{(s)} &= (r - r_+)^{-\frac{\delta_s(\omega)}{2}} u_0^{(s)} + \dots, & s &= \frac{3}{2}, -\frac{1}{2}, \\ u^{(s)} &= (r - r_+)^{\frac{\delta_s(\omega)}{2}} u_0^{(s)} + \dots, & s &= \frac{1}{2}, -\frac{3}{2}, \end{aligned} \quad (10)$$

where the  $u_0^{(s)}$  are four arbitrary complex coefficients. Thus the ingoing boundary condition at the horizon is given by  $u_0^{(1/2)} = u_0^{(-3/2)} = 0$ . The residual gauge transformations allows us to gauge away either  $u_0^{(-1/2)}$  or  $u_0^{(3/2)}$ . The situation for  $T = 0$  is slightly more subtle and is explained in [15]. It can be shown that if  $(u^{(-s)})^*$  is a solution then so is  $u^{(s)}$ . This is related to the action of time reversal and will be useful below.

Having solved the gravitino equations in this way, one can rewrite  $\psi_\mu$  in terms of  $u^{(s)}$  and  $v^{(s)}$ . compare with the asymptotic expansion, and obtain the boundary data  $a_\alpha$  and  $b_\alpha$ . We then obtain  $t_{11}$  from (9).

## RESULTS

We now summarise some results for the spectral function based on solving the ODEs numerically.

As illustrated in figure 1, the most prominent feature for  $T \neq 0$  is the large peak near  $\tilde{\omega} = 0$  associated with a pole of  $G_{\alpha\beta}$  at  $(\tilde{\omega}, k) = (0, 0)$ . This long-wavelength Goldstino pole has been discussed before in a hydrodynamical context. In supersymmetric theories in addition to ordinary sound waves there are weakly damped propagating “super-sound” waves, or “phoninos” [11, 12]. At  $\mu = 0$ , this gives a pole in  $G_{\alpha\beta}$  at  $(\omega, k) = (0, 0)$  [12], and was analysed holographically in [10].

In our case  $\mu \neq 0$ , and the weakly gauged  $R$ -symmetry means the phonino pole is shifted to  $(\tilde{\omega}, k) = (0, 0)$ , exactly as in figure 1. For higher values of  $k$ , this peak disappears. At the same time the spectral weight gets redistributed to positive  $\omega$ , where a bump develops. At

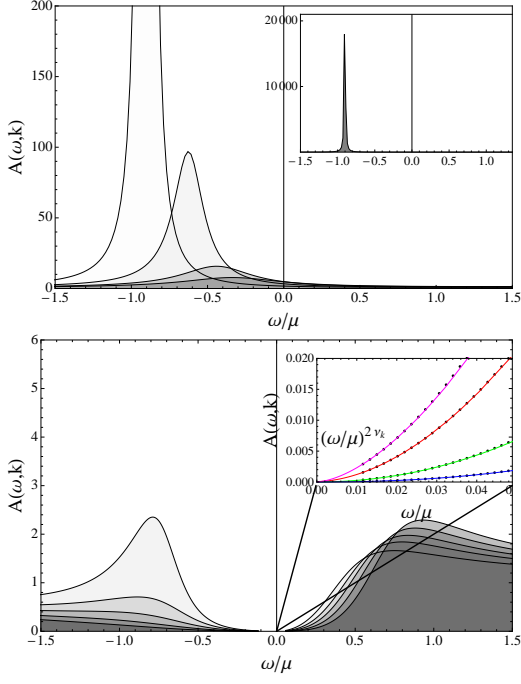


FIG. 1: The spectral function  $A(\omega, k)$ . The top panel is for  $T/\mu = 0.44$  and momenta  $k\ell \in (0.1, 1.1)$ , with larger values of  $k$  in darker shades of grey. The bottom panel is for  $T = 0$  and  $k\ell \in (1.2, 2.1)$ .

low temperatures and small  $\omega$  there is a region of the order of the chemical potential, where the density of states is depleted. An analogous feature was interpreted in [13] as a hard (Mott) gap.

Some results for the spectral function for  $T = 0$  are shown in figure 1. The phonino pole is still present at  $(\tilde{\omega}, k) = (0, 0)$ , much as in the top panel. We also see that the spectral function vanishes at  $\omega = 0$  for all values of  $k$ . In fact there is a scaling of the form  $A \propto \omega^{2\nu_k}$  corresponding to a soft power-law gap.

We can derive this behaviour analytically. Indeed, by the method of matched asymptotic expansions, as in [4], at  $T = 0$  and at leading order in  $\omega$  we can show

$$t_{11}(\omega, k) = t_{11}(0, k) (1 + C(k)\mathcal{G}(\omega, \nu_k) + \dots), \quad (11)$$

where

$$\mathcal{G}(\omega, \nu_k) = e^{-i\pi\nu_k} \frac{\Gamma(-2\nu_k)}{\Gamma(2\nu_k)} \frac{\Gamma(-1 - \frac{i}{2\sqrt{3}} + \nu_k)}{\Gamma(-1 - \frac{i}{2\sqrt{3}} - \nu_k)} (2\omega L_{(2)})^{2\nu_k} \quad (12)$$

with  $\nu_k = \sqrt{\frac{7}{12} + \frac{k^2}{2\mu^2}}$ . The function  $C(k)$  is independent of  $\omega$  and depends on the UV data of the system. Note that since  $\nu_k$  is real, for any  $k$ , there is no periodic log oscillatory behaviour as seen in the bottom-up models.

If  $t_{11}(0, k)$  is real then we can immediately extract the scaling relation for the spectral function

$$A(\omega, k) \propto \omega^{2\nu_k}, \quad (13)$$

for small  $\omega$ , exactly as we see in our numerical results. The reality of  $t_{11}(0, k)$  follows from the  $u^{(s)} \rightarrow (u^{(-s)})^*$  symmetry we mentioned above. Thus the vanishing of the spectral weight at  $\omega = 0$  and  $T = 0$  is not a hard Mott-like gap but rather a power-law, characteristic of a local massless sector of states associated with the  $AdS_2$  factor of the bulk near-horizon region.

Finally, it would be worthwhile to extend the results of this paper and [15] to study fermion spectral functions in the more involved top-down models of [18]. This would be particularly interesting as they include non-supersymmetric CFTs whose gravity duals are known to be perturbatively stable. Furthermore, it would be interesting to elucidate the impact of the superfluid phase [19, 20] at low temperatures and also to see whether the additional bulk fermions reveal any underlying Fermi surface.

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